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FAILURE OF THIN-WALLED MEMBERS UNDER PATCH LOADING AND SHEAR

by

M. Elgaaly, M.S.E., Sc.D., M.ASCE*

INTRODUCTION

Webs of thin-walled structural members can be subjected to a local inplane compressive patch loading. The web panel will rarely be subjected to this discrete edge loading only, if it is an end panel it will be subjected to an additional in-plane shear and if a central panel it will be subjected to an additional in-plane bending moment.

The compressive patch load which will cause buckling of a rectangular plate is given by,

$$\frac{P_{cr}}{bt} = K \frac{\pi^2 D}{d^2 t} \tag{1}$$

in which P_{cr} = critical edge load; b, d, and t = width, depth and thickness of the plate, respectively (Figure 1a); K = a non-dimensional buckling coefficient; and D = flexural rigidity of the plate. The buckling coefficient K is function of the relative length of the loaded patch $\beta = \frac{c}{b}$, the aspect ratio of the panel $\alpha = \frac{b}{d}$ and the panel's boundary support conditions. In Ref. (1), the buckling coefficient K is given for simply supported panels with aspect ratio 1, 1.5 and 2; for various values of β.

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The presence of either an additional shear or moment will reduce the applied edge load necessary to buckle the panel. Interaction curves are given in Ref. (1) for the following cases: (1) a simply supported square plate subjected to a combination of uniform edge-loading and in-plane bending moment or uniform shear load and (2) a simply supported square plate subjected to a combination of a discrete edge load (β = 0.2) and in-plane bending moment or uniform shear load.

In slender web panels adequately supported the failure load exceeds the buckling load. Theoretical determination of the ultimate strength is a complicated matter, involving nonlinearities of geometry and material. Results from an extensive experimental study of the failure of web panels when subjected to a central patch load as shown in Figure 1a have been presented in Ref. (1). In the same reference the following emperical formula was given to determine the failure load of a web panel loaded as shown in Figure 1a,

$$P_u = P_{cr} \left[4.5 + 6.4 \left(\frac{c}{b}\right)\right] \frac{d}{t} \times 10^{-3} \qquad (2)$$

in which P_u = ultimate edge load and the rest of the symbols are as defined before. Equation (2) gives reasonably accurate values of the ultimate load (P_u) for panels with slenderness ratio up to about 250 made of mild structural steel of a yield stress about 16 t/in^2 (36 ksi).

The work was extended to study the behavior of web panels under discrete edge compressive loading and in-plane bending, as shown in Figure 1b, up to failure; Ref. (2). The purpose was to develop a simple method of estimating the ultimate carrying capacity under this loading condition. A conservative relationship between the ratios $\frac{P_u}{P_{uo}}$ and $\frac{M_u}{M_{uo}}$ was established and is given in Eq. (3),

$$\left(\frac{P_u}{P_{uo}}\right)^3 + \left(\frac{M_u}{M_{uo}}\right)^3 = 1.0 \qquad (3)$$

in which P_u and P_{uo} = ultimate patch load in the presence and absence of bending respectively and M_u and M_{uo} = ultimate in-plane bending moment in the presence and absence of the patch load, respectively.

The present paper presents the results obtained from an experimental study conducted to investigate the ultimate carrying capacity of web panels subjected to patch loading and in-plane uniform shear, as shown in Figure 1c.

Test Specimens

The primary purpose of the test program was to determine the ultimate load strength characteristics of the webs of cold formed steel structural members when subjected to in-plane discrete compressive loading together with uniform shear. The general details of the test specimens are shown in Figure 2. The section used is a hat-section with slightly inclined webs. Two thicknesses were used, namely, .037" and .06" which correspond to web slenderness ratios of 325 and 200, respectively. Six strong diaphragms were fitted and bolted to the specimens as shown in Figures 2a and 2b. The two central diaphragms are to avoid local failure under the central load P_s, the two diaphragms over the supports to avoid local failure from the reactive forces, and the two outside diaphragms were used to provide the 4" x 12" exterior panels to ensure the ultimate carrying capacity of the test panel under shear loading by serving as an anchor for the tension field stress. To exclude the possibility of flange failure prior to web failure a central longitudinal stiffener was bolted to the flange as shown in Figure 2c.

The specimens were simply supported during testing and subjected to a central concentrated load P_s. The test panels, on each side of the central load P_s, were subjected (one at a time) to a central patch load P of relative width $\frac{c}{b} = 0.2$, in addition to the shearing force $Q = .5P_s + .27P$, as shown in Figure 3. The variation in the magnitude of the shearing force Q was achieved by varying the magnitude of P_s. The details of the eleven specimens tested are given in Table I. As can be noted from the table, most of the specimens were tested twice by applying the patch load P to each side of the specimen, one at a time, and in each time a different ratio of $\frac{P}{P_s}$ was used. This was planned such that the failure of the panel tested first will not affect the behavior of the specimen when testing the second panel.

Experimental Apparatus

The specimens were tested in a self-straining frame and the loads were applied by means of two push-type hydraulic jacks through a central roller directly to the specimen for the central load P_s and on to a loading plate which distributed it over a distance c = 2.4" for the patch load P; as shown in Figure 4. Two electric load cells were placed between the jacks and the rollers as can be noted in the figure. The test specimen was supported on hardened steel rollers which could be adjusted to ensure that the load was applied equally to both webs.

The lateral deflection of the web was recorded using the deflection recording apparatus shown in Figure 4; with the aid of this frame the five linear displacement transducers could be adjusted to any position, thus enabling an accurate recording of the deformed shape of the web to be obtained. Tests conducted prior to testing established that the accuracy of the device was such that repeatability of the readings was ensured. During testing the transducers were moved to 14 different locations so that the deflection of 70 points on the web were recorded. These transducers were connected to a data logger which printed out directly the values of the deflections in units of 0.001 inches.

Test Procedure

The loads were applied to the test specimens in small increments in the elastic range, and in smaller increments after yielding had begun. In the inelastic range, all plastic flow was allowed to take place at each load increment before any lateral deflection readings were taken. The load cells were calibrated before and after each test.

Since the specimens were from the same patch of sections used in previous testing programs reported in Ref. (1) and (2), it was assumed that the material properties are identical to those reported in Ref. (2). According to the results reported in that reference from several test coupons the material behaved in a manner typical of that expected for mild structural steel and the yield stresses were in the range of 15-18 tons/in^2.

Test Results

In the present paper the full test history will not be given, comments will be restricted to presenting and discussing typical overall behavior together with a detailed study of the ultimate load behavior of the panels under patch loading and shear.

The mode of failure under patch loading only (tests 201 and 203) was typical of that obtained earlier and reported in Ref. (1). In this case, failure occurred by the formation of a local yield curve which corresponds closely to a segment of a circle and has a width equal to that of the patch load. The ultimate patch load P_u obtained from the tests reported here are slightly higher than the corresponding values reported in Ref. (1); P_u = 0.383 tons compared to 0.370 tons reported in Ref. (1) for $\frac{d}{t}$ = 325 and 0.851 tons compared to 0.84 tons reported in Ref. (1) for $\frac{d}{t}$ = 200. This increase could be due to the fact that the vertical edges of the panel in this series are partially fixed due to the continuity on both sides rather than simply supported as for the panels reported if Ref. (1).

The ultimate carrying capacity of the panels under pure shear is given by tests 2111 and 2113. The typical mode of failure under this loading condition is shown in Figure 5. From a review of the so-called incomplete diagonal tension engineering theories for ultimate strength under pure shear (3), the so-called true Basler theory (4,5) is applicable in our case since the flexural rigidity of the flanges are negligible. In this case the ultimate shear stress τ_u is given by,

$$\tau_u = \tau_{cr} + \frac{\sigma_{yw} - \sqrt{3}\,\tau_{cr}}{2(\sqrt{1+\alpha^2} + \alpha)} \tag{4}$$

where τ_{cr} = critical buckling stress,

σ_{yw} = yield stress of the web material, and

α = aspect ratio of the panel.

Since the vertical edges of the panel in the tests reported here are somewhere between simply supported and clamped and the yield stress of the web material is somewhere between 15 and 18 t/in^2, τ_u was calculated from Eq. (4) for the limiting conditions and the results are tabulated in Table II together with the corresponding experimental results.

As can be noted the experimental values are within the range of the predicted values using Eq. (4).

The primary purpose of the tests reported in this paper was to determine the influence which the additional shearing force would have upon the capacity of the web to withstand the patch loading. The test results are given in Table III. The patch load P and the shearing force Q as quoted throughout this paper refer to the loads and forces acting on each web. A typical mode of failure under patch loading and shear as encountered in these tests is shown in Figure 6 for test number 281. In what follows the ultimate patch load in the absence of the shear, P_{uo}, and the ultimate shearing force in the absence of the patch load, Q_{uo}; are used as datums. The ratios of the actual ultimate load P_u to P_{uo} and the corresponding shearing force Q_u to Q_{uo} are given in Table III. These values are plotted in Figure 7; a conservative relationship between $\frac{P_u}{P_{uo}}$ and $\frac{Q_u}{Q_{uo}}$ is given in Eq. (5),

$$\left(\frac{P_u}{P_{uo}}\right)^{1.8} + \left(\frac{Q_u}{Q_{uo}}\right)^{1.8} = 1.0 \qquad (5)$$

The interaction curve given by Eq. (5) is plotted in Figure 7.

Post Buckling Strength

The presence of a shearing force Q will reduce the applied patch load necessary to buckle the panel. The interaction curve for patch loading ($\frac{c}{b}$ = 0.2) and shear, taken from Ref. (1), for a panel simply supported along its edges is shown in Figure 8. The panels considered in this paper are, in fact, simply supported along their horizontal edges and partially clamped along their vertical edges, however, when calculating the critical buckling loads it will be assumed that the panels are simply supported along all four edges. In this case,

$$P_{cro} = 3.45 \frac{\pi^2 E}{12(1 - \upsilon^2)} \frac{bt}{(d/t)^2} \tag{6.1}$$

$$Q_{cro} = 9.34 \frac{\pi^2 E}{12(1 - \upsilon^2)} \frac{dt}{(d/t)^2} \tag{6.2}$$

and $$Q_{cr} = \gamma P_{cr} \tag{6.3}$$

where γ is the coefficient given in the last column of Table I.

From Eq. (6), it can be proved that

$$\frac{P_{cr}}{P_{cro}} = \frac{2.707}{\gamma} \frac{Q_{cr}}{Q_{cro}} \tag{7}$$

Equation (7) represents the equation of a straight-line and is plotted in Figure 8 for all the panels tested. The intersections of these straight-lines with the interaction curve give the reduced (due to the presence of the shear) values of P_{cr}.

To demonstrate the post-buckling strength of the panels tested, the values of $\frac{P_u}{P_{cr}}$ are given in Table IV and are shown in Figure 9 for various values of γ and for $\frac{d}{t}$ = 325 and 200. As can be noted from the figure the post-buckling strength is higher for more slender webs. The values of $\frac{P_u}{P_{cr}}$ increase as γ increases. The increase in $\frac{P_u}{P_{cr}}$ is, for all practical purposes, a linear function of γ. However, at a certain value of γ, the increase ceases and $\frac{P_u}{P_{cr}}$ reaches an optimum value. It is of interest to note that the optimum value of $\frac{P_u}{P_{cr}}$ is nearly equal to $\frac{Q_{uo}}{Q_{cro}}$ which is the post-buckling strength under pure shear loading.

Summary and Conclusions

A test program to study the effect of the presence of in-plane shear on the ultimate capacity of web panels under in-plane discrete edge compressive loading was described. The results from the tests have shown that the presence of the shear will reduce the ultimate edge load carrying capacity of the web. An approximate relationship between the ratios $\frac{P_u}{P_{uo}}$ and $\frac{Q_u}{Q_{uo}}$ has been established.

It has been shown that the post-buckling strength of the panels under discrete edge compressive loading will increase with the increase in the slenderness ratio of the panel. Moreover, panels subjected to the combination of discrete edge compressive loading and shear will exhibit higher post-buckling strength than panels subjected to the discrete edge compressive loading only.

Acknowledgement

The study reported herein was carried out by the author during his stay in University College, Cardiff, U.K., as a lecturer in the department of Civil and Structural Engineering. The support of Professor K. C. Rockey, the head of the department in Cardiff, is gratefully acknowledged.

APPENDIX I

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APPENDIX II

NOTATIONS

The following symbols are used in this paper:

b = width of web panel,

c = width of patch load,

D = flexural rigidity of web panel,

d = depth of web panel,

E = Young's modulus of elasticity,

K = non-dimensional critical buckling coefficient,

M = in-plane bending moment,

M_u = ultimate moment in the presence of patch load,

M_{uo} = ultimate moment in the absence of patch load,

P = patch load,

P_{cr} = critical buckling load,

P_s = central concentrated load,

P_u = ultimate patch load in the presence of shear or bending,

P_{uo} = ultimate patch load in the absence of shear and bending,

Q = shear force,

Q_{cr} = critical buckling shear force,

Q_u = ultimate shear force in the presence of patch load,

Q_{uo} = ultimate shear force in the absence of patch load,

t = thickness of web panel,

α = aspect ratio of web panel,

β = relative width of patch load,

γ = a coefficient equals $\frac{Q}{P}$,

τ_{cr} = critical buckling shear stress,

τ_u = ultimate shear stress,

υ = Poisson's ratio, and

σ_{yw} = yield stress of web material.

Specimen Number	Test Number	Thickness (inches)	P_s	Q
1	201	0.037	0	0.27P
2	211	0.037	1.1P	0.82P
1	221	0.037	2.0P	1.27P
3	231	0.037	2.9P	1.72P
4	241	0.037	3.9P	2.22P
5	251	0.037	5.1P	2.82P
4	261	0.037	8.1P	4.32P
5	271	0.037	9.9P	5.22P
2	281	0.037	15.8P	8.17P
3	291	0.037	19.3P	9.92P
6	2101	0.037	37.2P	18.87P
7	2111	0.037	P_s (only)	$0.5P_s$
8	203	0.06	0	0.27P
8	233	0.06	3.1P	1.82P
9	263	0.06	7.2P	3.87P
10	273	0.06	9.9P	5.22P
9	283	0.06	14.3P	7.42P
11	2113	0.06	P_s (only)	$0.5P_s$

TABLE I

Details of Test Specimens

	t = .037"		t = .06"	
	yield stress		yield stress	
	$\sigma_y = 15\ t/in^2$	$\sigma_y = 18\ t/in^2$	$\sigma_y = 15\ t/in^2$	$\sigma_y = 18\ t/in^2$
Simply supported along four sides	3.773	4.394	4.859	5.480
Clamped along two parallel sides and simply supported along the others	3.985	4.606	5.415	6.037
Experimental Value	4.552		5.171	

TABLE II

Comparison Between the Experimental Results for the Case of Pure Shear and the So-Called True Basler Theory

Test Number	$\frac{d}{t}$	P_u (tons)	$\frac{P_u}{P_{uo}}$	Q_u (tons)	$\frac{Q_u}{Q_{uo}}$
201	325	0.383	1.000	0.103	0.051
211	325	0.383	1.000	0.314	0.155
221	325	0.383	1.000	0.486	0.241
231	325	0.351	0.917	0.604	0.299
241	325	0.351	0.917	0.779	0.386
251	325	0.319	0.833	0.900	0.445
261	325	0.285	0.743	1.231	0.609
271	325	0.266	0.694	1.389	0.687
281	325	0.214	0.560	1.748	0.865
291	325	0.187	0.488	1.855	0.918
2101	325	0.103	0.268	1.944	0.962
2111	325	0.000	0.000	2.021	1.000
203	200	0.851	1.000	0.230	0.062
233	200	0.792	0.931	1.441	0.387
263	200	0.649	0.763	2.512	0.675
273	200	0.538	0.632	2.808	0.754
283	200	0.427	0.501	3.168	0.851
2113	200	0.000	0.000	3.723	1.000

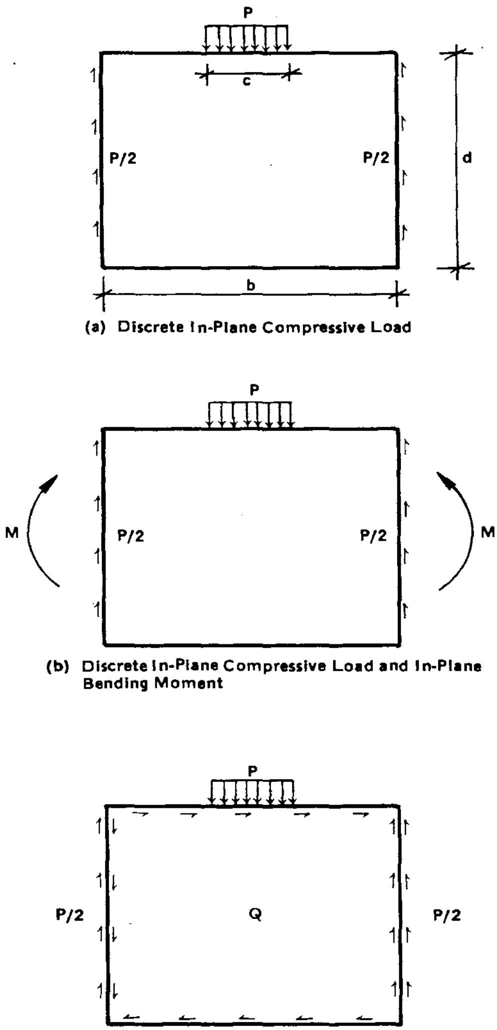
TABLE III

Test Results

Test Number	$\frac{d}{t}$	γ	Reduction Factor for Reduced P_{cr}	$\frac{P_u}{P_{cr}}$
201	325	0.27	.992	2.271
211	325	0.82	.943	2.389
221	325	1.27	.880	2.560
231	325	1.72	.807	2.558
241	325	2.22	.730	2.828
251	325	2.82	.640	2.932
261	325	4.32	.480	3.493
271	325	5.22	.415	3.770
281	325	8.17	.282	4.464
291	325	9.92	.240	4.583
2101	325	18.87	.135	4.488
203	200	0.27	.992	1.180
233	200	1.82	.787	1.384
263	200	3.87	.505	1.768
273	200	5.22	.415	1.783
283	200	7.42	.307	1.913

TABLE IV

Post-Buckling Strength



(a) Discrete In-Plane Compressive Load

(b) Discrete In-Plane Compressive Load and In-Plane Bending Moment

(c) Discrete In-Plane Compressive Load and In-Plane Shear

FIG. 1 - DIMENSIONS OF THE PANEL AND CASES OF LOADING

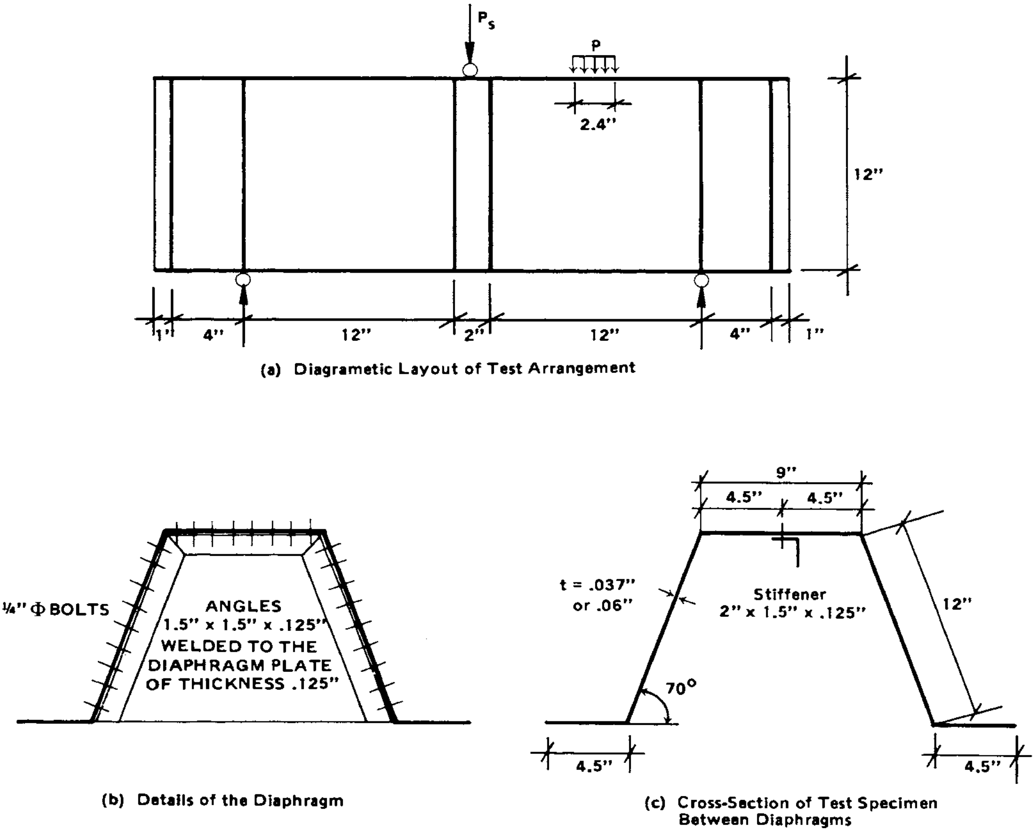


FIG. 2 - GENERAL DETAILS OF TEST SPECIMEN

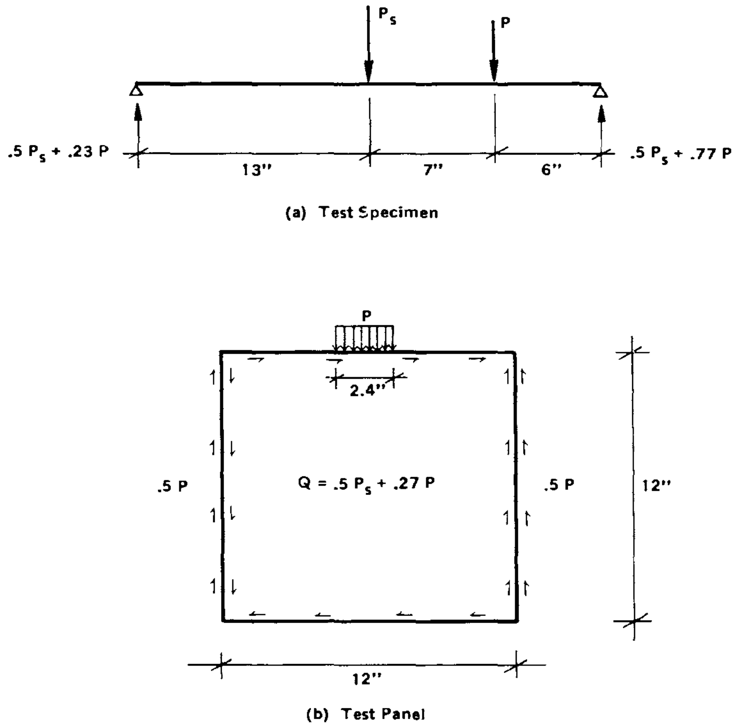


FIG. 3 - LOADING DIAGRAMS

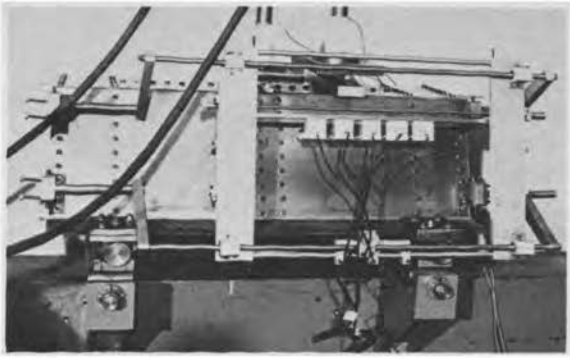


FIG. 4 - TEST SET-UP

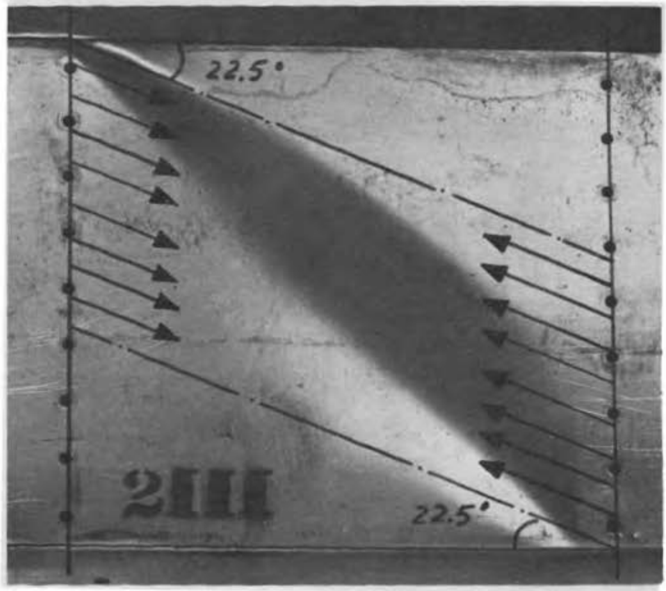


FIG. 5 - MODE OF FAILURE UNDER PURE SHEAR

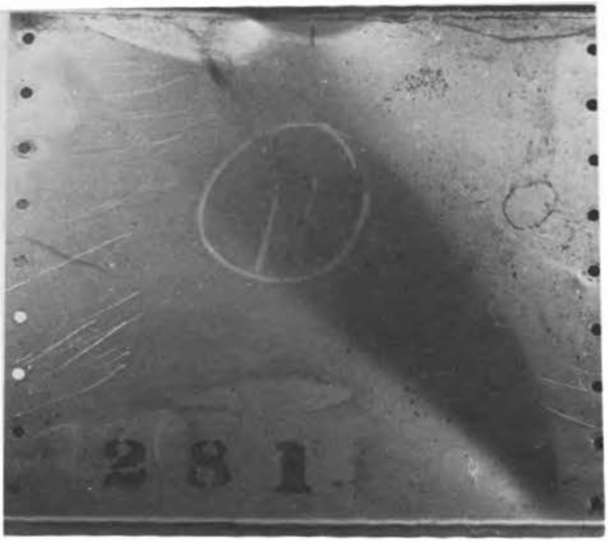


FIG. 6 - TYPICAL MODE OF FAILURE UNDER PATCH LOADING AND SHEAR

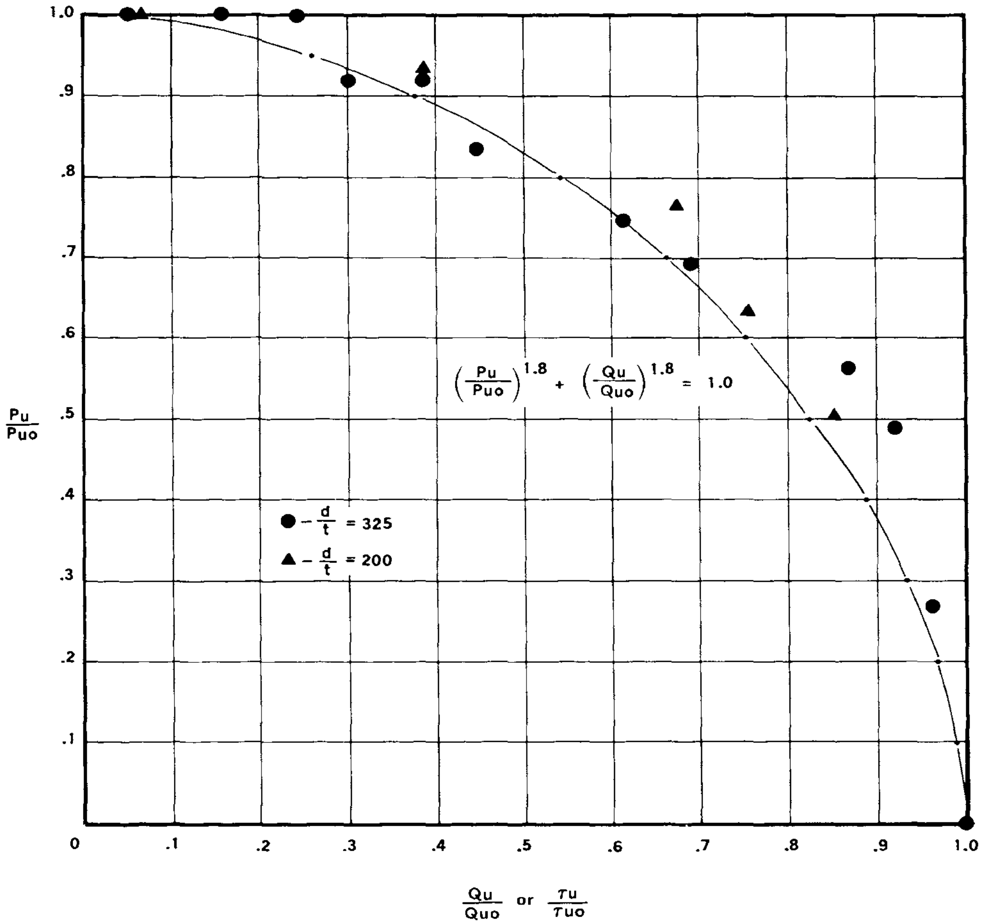


FIG. 7 - RELATION BETWEEN $\frac{Pu}{Puo}$ and $\frac{Qu}{Quo}$

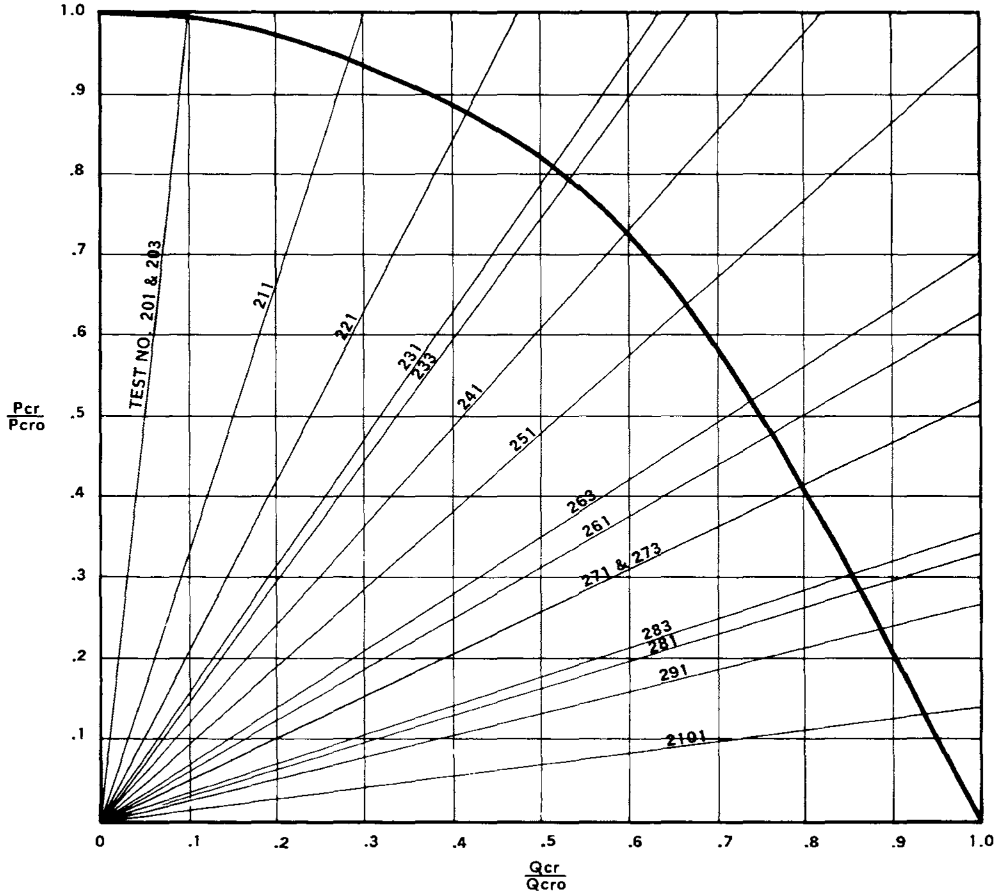


FIG. 8 - INTERACTION CURVE FOR PATCH LOADING AND SHEAR, $\frac{c}{b} = 0.2$

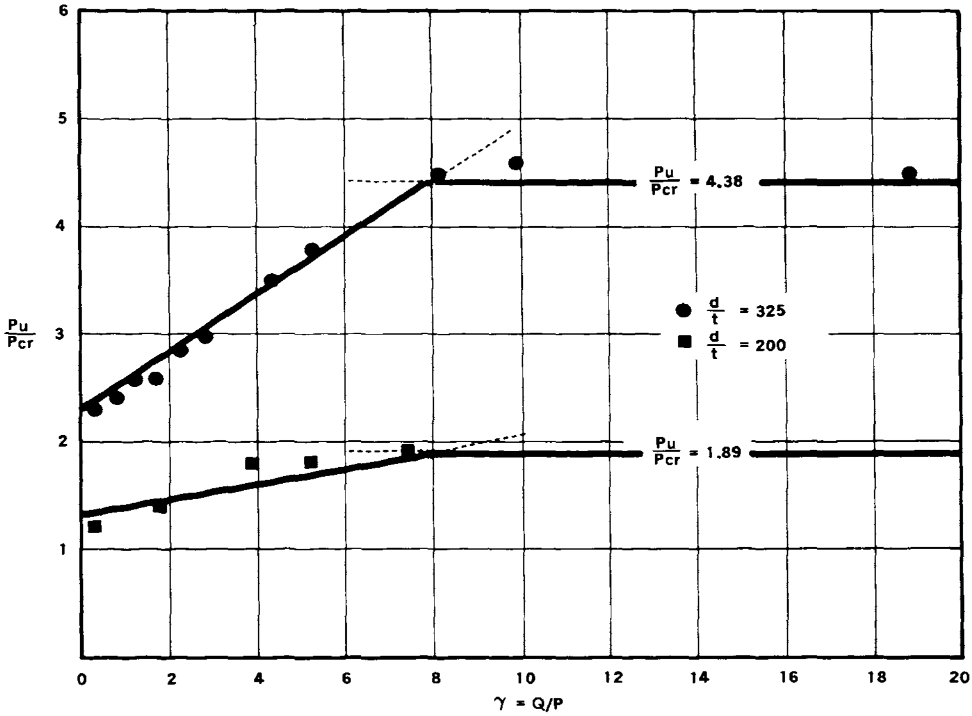


FIG. 9 - VARIATION OF $\frac{Pu}{Pcr}$ WITH γ